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Heat transfer in laminar flow of non-Newtonian fluids in ducts of elliptical section

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Abstract

Laminar forced convection inside tubes of various cross-section shapes is of interest in the design of a low Reynolds number heat exchanger apparatus. Heat transfer to thermally developing, hydrodynamically developed forced convection inside tubes of simple geometries such as a circular tube, parallel plate, or annular duct has been well studied in the literature and documented in various books, but for elliptical duct there are not much work done. The main assumption used in this work is a laminar flow of a power flow inside elliptical tube, under a boundary condition of first kind with constant physical properties and negligible axial heat diffusion (high Peclet number). To solve the thermally developing problem, we use the generalized integral transform technique (GITT), also known as Sturm–Liouville transform. Actually, such an integral transform is a generalization of the finite Fourier transform where the sine and cosine functions are replaced by more general sets of orthogonal functions. The axes are algebraically transformed from the Cartesian coordinate system to the elliptical coordinate system in order to avoid the irregular shape of the elliptical duct wall. The GITT is then applied to transform and solve the problem and to obtain the once unknown temperature field. Afterward, it is possible to compute and present the quantities of practical interest, such as the bulk fluid temperature, the local Nusselt number and the average Nusselt number for various cross-section aspect ratios.

Keywords: Non-Newtonian fluids; Power law fluids; Forced convection; Integral transform; Elliptical tube

1. Introduction

Several works have been dedicated to problems of internal forced convection, particularly those that characterize extensions of the classic Graetz problem. On the several methods and solution techniques found in the literature, it is observed that, for this class of diffusive problems, the most accurate and more powerful are generally applied to the simplest problems, where the results are already known. On the other hand, analytical solutions are few frequents for problems of flow in ducts with irregular geometry submitted to more complex boundary conditions or for the cases where the properties of the fluid present some dependence with the temperature. In particular, problems that involve processing non Newtonian fluids characterize research field that has many applications. However, there are also few solutions for problems involving the flow of non-Newtonian fluids.

Recently, a generalization of the integral transform [2] has been developed to obtain solutions for several complex diffusion problems, specially the ones that do not posses a closed form solution by the method of the separation of variables. It is the Generalized Integral Transform Technique – GITT [3], a method which has been used successfully to solve several diffusive problems such as those dealing with the flow in ducts with irregular geometries [4,5], with time varying coefficients and problems involving space dependence for the boundary conditions [6,7], problems with thermally and hydrodynamically developing flows [8,9], diffusive problems involving moving boundaries [10,11], problems of non-Newtonian fluid flows [12,13], among others.

In this study, the calculation of heat transfer parameters of a power-law fluid in thermally developing flow, inside straight

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Nomenclature

a, b	major and minor ellipsis semi-axis	m
A_{ij}	equation coefficient, Eq. (27)	
B_{ijmn}	equation coefficient, Eq. (35)	
c_p	specific heat	$\mathrm{J\,kg^{-1}\,K^{-1}}$
D_h	hydraulic diameter	m
h	heat transfer coefficient	$\mathrm{W\,m^{-2}\,K^{-1}}$
H	equation coefficient, Eq. (15)	
J	Jacobian determinant, Eq. (16)	
k	fluid conductivity	$\mathrm{W\,m^{-1}\,K^{-1}}$
K_i, Z_m	normalized eigenfunctions, Eq. (24) and (33a)	
L_{th}	thermal entry length, Eq. (45)	
n	flow behavior index	
N_i, M_m	normalization integrals, Eq. (25) and (33b)	
$Nu, \overline{Nu}$	local and average Nusselt numbers, Eq. (40) and (44)	
P	perimeter of the elliptical contour	m
Pe	Peclet number, Eq. (7e)	
Pr	Prandtl number	
q''	heat flux at the contour	$\mathrm{W\,m^{-2}}$
T, T_{av}	temperature and average temperature	K
T_w	wall temperature	K
T_0	constant inlet temperature	K
Re	Reynolds number	
V	velocity field	$\mathrm{m\,s^{-1}}$
V_{av}	flow average velocity	$\mathrm{m\,s^{-1}}$
u, v	dimensionless elliptical coordinates	
U	dimensionless velocity field, Eq. (7d)	
v_0	coordinate contour, Eq. (12b)	
x, y, z	coordinate axes	m
X, Y, Z	dimensionless coordinates, Eq. (7b)	

Greek symbols

α, β	dimensionless ellipsis semi-axes, Eq. (7c)	
α^*	dimensionless focal distance, Eq. (12a)	
ρ	fluid density	$\mathrm{kg\,m^{-3}}$
θ	dimensionless temperature, Eq. (7a)	
$\bar{\theta}, \tilde{\bar{\theta}}$	transformed potentials	
θ_{av}	dimensionless average temperature, Eq. (38)	
ϕ_m	eigenfunction related to the coordinate v, Eq. (30b)	
λ_m	eigenvalue associated to the eigenfunction $\phi_m(v)$, Eq. (30a)	
ψ_i	eigenfunction related to the coordinate u, Eq. (21b)	
μ_i	eigenvalue associated to the eigenfunction $\psi_i(u)$, Eq. (21a)	
Ω, Γ	cross section domain and cross section contour	

Subscripts

i, j, m, n integer indices

Superscripts

$-$	transform operator related to the coordinate u
$\sim$	transform operator related to the coordinate v

duct of elliptical cross section is performed. The typical difficulty for obtaining an analytical solution for this problem by means of the well known classical techniques resides in the impossibility of the separation of variables. With regard to the application of the boundary conditions there is also an additional difficulty due to the non regular two-dimensional characteristic of the cross section of the elliptical duct. In particular, for the solution of this problem, a convenient change of variables is employed for the determination of the velocity profiles inside the elliptical duct and also, to transform the elliptical profile into a new geometry in order to simplify the application of the boundary conditions. The GITT is then applied to the energy equation for this problem to obtain the temperature field in the flow and, consequently, to determine the parameters of interest, such as the average temperature and Nusselt number for various cross-section aspect ratios and fluid behavior index.

2. Analysis

In the formulation of the present problem, thermally developing and hydrodynamically developed laminar steady flow is assumed. The fluid enters the tube with a uniform temperature profile, the effects of the viscous dissipation were ignored, as well as the axial conduction, and the fluid properties were considered constants in the whole domain. Therefore, the energy equation, according to the coordinate system presented in Fig. 1(a), is given by

$$\rho c_p V(x,y)\frac{\partial T(x,y,z)}{\partial z} = k\left[\frac{\partial^2 T(x,y,z)}{\partial x^2} + \frac{\partial^2 T(x,y,z)}{\partial y^2}\right] \quad \{(x,y)\in\Omega,\ z>0\} \tag{1}$$

The velocity profile $V(x,y)$ for a fully developed flow of the power law fluid in a duct of elliptical cross section is given by [14]

$$V(x,y) = \frac{3n+1}{n+1}\left[1-\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^{\frac{n+1}{2n}}\right]V_{\mathrm{av}} \tag{2}$$

where n is the fluid behavior index and V_{av} is the flow average velocity.

The elliptical boundary condition analyzed deal with situations where the internal thermal resistance is larger than the external thermal resistance including the wall thermal resistance. Therefore, the temperature condition prescribed at the boundary for the present problem is

$$T(x,y,z) = T_w = \text{constant}, \quad \{(x,y)\in\Gamma\} \tag{3}$$

The others boundary conditions and the condition at the entrance of the problem can be written as:

$$\frac{\partial T(x,y,z)}{\partial x} = 0, \quad \{x=0,\ 0<y<b,\ z>0\} \tag{4}$$

$$\frac{\partial T(x,y,z)}{\partial y} = 0, \quad \{0<x<a,\ y=0,\ z>0\} \tag{5}$$

$$T(x,y,0) = T_0, \quad \{(x,y)\in\Omega\} \tag{6}$$

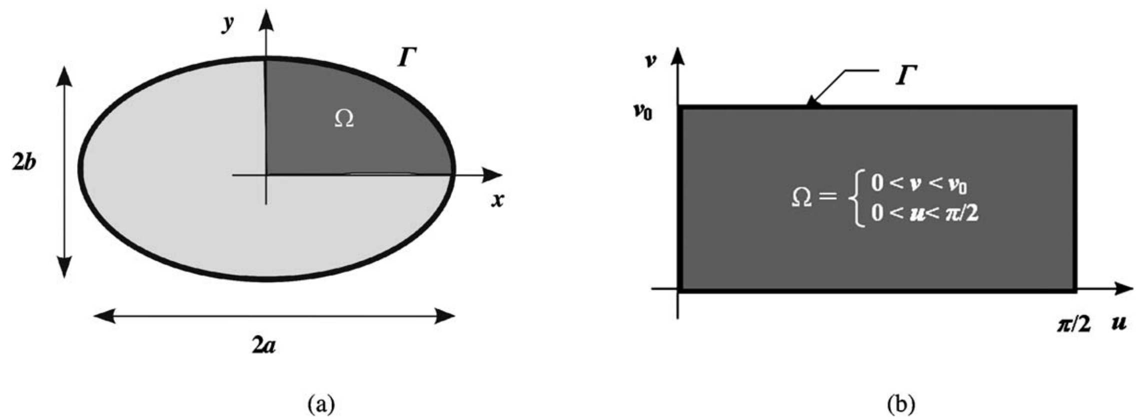

Fig. 1. (a) Geometry of the duct cross section and coordinate system and (b) geometry in the new coordinate system.

2.1. Transformation of coordinates for the thermal problem

The temperature profile and other physical and geometrical parameters are written in dimensionless form as

$$\theta(X,Y,Z)=\frac{T(X,Y,Z)-T_w}{T_0-T_w} \tag{7a}$$

$$X=\frac{x}{D_h},\qquad Y=\frac{y}{D_h},\qquad Z=\frac{z}{D_h Pe} \tag{7b}$$

$$\alpha=\frac{a}{D_h},\qquad \beta=\frac{b}{D_h} \tag{7c}$$

$$U(X,Y)=\frac{V(x,y)}{V_{av}} \tag{7d}$$

$$Pe=\frac{\rho c_p V_{av} D_h}{k} \tag{7e}$$

As a consequence of the dimensionless parameters defined above, the energy equation can be rewritten as

$$U(X,Y)\frac{\partial\theta(X,Y,Z)}{\partial Z}=\frac{\partial^2\theta(X,Y,Z)}{\partial X^2}+\frac{\partial^2\theta(X,Y,Z)}{\partial Y^2} \tag{8}$$

where the dimensionless velocity profile is obtained from Eqs. (2) and (7d)

$$U(X,Y)=\frac{3n+1}{n+1}\left[1-\left(\frac{X^2}{\alpha^2}-\frac{Y^2}{\beta^2}\right)^{\frac{n+1}{2n}}\right] \tag{9}$$

and the inlet and boundary conditions are given by

$$\theta(X,Y,Z)=1,\quad \big\{(X,Y)\in\Gamma,\ Z=0\big\} \tag{10a}$$

$$\theta(X,Y,Z)=0,\quad \big\{(\mathrm{X},\mathrm{Y})\in\Gamma,\ Z>0\big\} \tag{10b}$$

$$\frac{\partial\theta(X,Y,Z)}{\partial X}=0,\quad \{0\leqslant X\leqslant\alpha,\ Y=0,\ Z>0\} \tag{10c}$$

$$\frac{\partial\theta(X,Y,Z)}{\partial Y}=0,\quad \{X=0,\ 0\leqslant Y\leqslant\alpha,\ Z>0\} \tag{10d}$$

The orthogonal system of elliptic coordinates (u,v) is used to transform the original domain (elliptical contour in plane (x,y)) into a domain with rectangular contour as presented in Fig. 1(b):

$$X=\alpha^*\cosh(v)\cos(u) \tag{11a}$$

$$Y=\alpha^*\sinh(v)\sin(u) \tag{11b}$$

with:

$$\alpha^*=\frac{\alpha}{\cosh(v_0)} \tag{12a}$$

$$v_0=\operatorname{arc\,tanh}\left(\frac{\beta}{\alpha}\right),\quad \{0<\beta<\alpha\} \tag{12b}$$

With these new variables the equations of the elliptical contour become:

$$\left[\frac{X}{\alpha^*\cosh(v_0)}\right]^2+\left[\frac{Y}{\alpha^*\operatorname{senh}(v_0)}\right]^2=1 \tag{13}$$

and the energy equation:

$$H(u,v)\frac{\partial\theta(u,v,Z)}{\partial Z}=\frac{\partial^2\theta(u,v,Z)}{\partial u^2}+\frac{\partial^2\theta(u,v,Z)}{\partial v^2} \tag{14}$$

with, $H=H(u,v)$ given by:

$$H(u,v)=J\left(\frac{X,Y}{u,v}\right)U(u,v) \tag{15}$$

where J is the matrix Jacobian for transformation of the coordinates in the plane (X,Y) to the coordinates in the plane (u,v):

$$J=\frac{\partial(X,Y)}{\partial(u,v)}=\alpha^{*^2}\big[\sinh^2(v)+\sin^2(u)\big] \tag{16}$$

and $U(u,v)$ is the velocity profile in variables (u,v):

$$U(u,v)=\frac{3n+1}{n+1}\left[1-\left(\frac{\alpha^{*2}}{\alpha^2}\cosh^2(v)\cos^2(u)-\frac{\alpha^2}{\beta^2}\sinh^2(v)\sin^2(u)\right)^{\frac{n+1}{2n}}\right] \tag{17}$$

The inlet and boundary conditions in the new coordinate system become

$$\theta(u,v,Z)=1,\quad \big\{(u,v)\in\Omega,\ Z=0\big\} \tag{18a}$$

$$\frac{\partial\theta(u,v,Z)}{\partial u}=0,\quad \{u=0,\ 0\leqslant v\leqslant v_0, Z>0\} \tag{18b}$$

$$\frac{\partial\theta(u,v,Z)}{\partial u}=0,\quad \{u=\pi/2,\ 0\leqslant v\leqslant v_0,\ Z>0\} \tag{18c}$$

$$\frac{\partial\theta(u,v,Z)}{\partial v}=0,\quad \{0\leqslant u\leqslant\pi/2,\ v=0,\ Z>0\} \tag{18d}$$

$$\theta(u,v,Z)=0,\quad \{0\leqslant u\leqslant\pi/2,\ v=v_0,\ Z>0\} \tag{18e}$$

2.2. Application of the GITT

To obtain the solution of the diffusion equation in the new coordinate system, Eq. (14), subjected to the conditions given by Eqs. (18), the GITT is then applied. In order to accomplish this, consider the following auxiliary eigenvalue problem:

$$\frac{d^2\psi(u)}{du^2}+\mu^2\psi(u)=0,\quad \{0\leqslant u\leqslant\pi/2\} \tag{19}$$

with

$$\frac{d\psi(u)}{du}=0,\quad \{u=0\} \tag{20a}$$

$$\frac{d\psi(u)}{du}=0,\quad \{u=\pi/2\} \tag{20b}$$

The eigenvalues and the eigenfunctions associated to this problem are:

$$\mu_i=2(i-1),\quad i=1,2,3,\ldots \tag{21a}$$

$$\psi_i(u)=\cos(\mu_i u) \tag{21b}$$

The above eigenfunctions are orthogonal allowing the following pair inverse-transform:

$$\bar{\theta}_i(v,Z)=\int_0^{\pi/2}K_i(u)\theta(u,v,Z)\,dv,\quad \text{transform} \tag{22}$$

$$\theta(u,v,Z)=\sum_{i=1}^{\infty}K_i(u)\bar{\theta}_i(v,Z),\quad \text{inverse} \tag{23}$$

where, $K_i(u)$ are the normalized eigenfunctions given by:

$$K_i(u)=\frac{\psi_i(u)}{N_i^{1/2}} \tag{24}$$

$$N_i=\int_0^{\pi/2}\psi_i^2(u)\,du=\begin{cases}\pi/2, & i=1\\ \pi/4, & i>1\end{cases} \tag{25}$$

From the internal product of $K_i(u)$ and of $\theta(u,v,Z)$ with Eqs. (14) and (19), respectively, and by making use of the boundary conditions given by Eqs. (18) and (20), the following coupled system of partial differential equations of second order can be obtained:

$$\sum_{j=1}^{\infty}A_{ij}(v)\frac{\partial\bar{\theta}_j(v,Z)}{\partial Z}+\mu_i^2\bar{\theta}_i(v,Z)=\frac{\partial^2\bar{\theta}_i(v,Z)}{\partial u^2} \tag{26}$$

$$A_{ij}(v)=\int_0^{\pi/2}K_i(u)K_j(u)H(u,v)\,du \tag{27}$$

Let us now consider the following eigenvalue problem:

$$\frac{d^2\phi(v)}{dv^2}+\lambda^2\phi(v)=0,\quad \{0\leqslant v\leqslant v_0\} \tag{28}$$

with:

$$\frac{d\phi(v)}{dv}=0,\quad \{v=0\} \tag{29a}$$

$$\phi(v_0)=0,\quad \{v=v_0\} \tag{29b}$$

The eigenvalues and eigenfunctions for this new problem are:

$$\lambda_m=\frac{(2m-1)\pi}{2v_0},\quad m=1,2,3,\ldots \tag{30a}$$

$$\phi_m(v)=\cos(\lambda_m v) \tag{30b}$$

The eigenfunctions $\phi_m(v)$ are orthogonal and permit the development of the following inverse-transform pair:

$$\tilde{\bar{\theta}}_{im}(Z)=\int_0^{v_0}\int_0^{\pi/2}K_i(u)Z_m(v)\theta(u,v,Z)\,du\,dv,\ \text{transform} \tag{31}$$

$$\theta(u,v,Z)=\sum_{i=1}^{\infty}\sum_{m=1}^{\infty}K_i(u)Z_m(v)\tilde{\bar{\theta}}_{im}(Z),\quad \text{inverse} \tag{32}$$

$Z_m(v)$ are the normalized eigenfunctions given by

$$Z_m(v)=\frac{\phi_m(v)}{M_m^{1/2}} \tag{33a}$$

$$M_m=\int_0^{v_0}\phi_m^2(v)\,dv=\frac{v_0}{2} \tag{33b}$$

For the computation of the transformed temperature potential $\tilde{\bar{\theta}}_{im}(Z)$, the procedure is similar to that of the first eigenvalue problem. By applying the internal product of $Z_m(v)$ and also of $\bar{\theta}_i(u,Z)$ to Eqs. (26) and (28), respectively, and using the boundary conditions, Eqs. (18d), (18e) and (29a), (29b), the following coupled system of ordinary differential equations of first order is obtained:

$$\sum_{n=1}^{\infty}\sum_{j=1}^{\infty}B_{ijmn}\frac{d\tilde{\bar{\theta}}_{jn}(Z)}{dZ}+\left(\mu_i^2+\lambda_m^2\right)\tilde{\bar{\theta}}_{im}(Z)=0 \tag{34}$$

$$B_{ijmn}=\int_0^{v_0}Z_m(v)Z_n(v)A_{ij}(v)\,dv \tag{35}$$

Parameters B_{ijmn} can be integrated and, therefore, are known. The solution of this system of equations, when subjected to the transformed inlet condition,

$$\tilde{\bar{\theta}}_{im}(0)=\int_0^{v_0}\int_0^{\pi/2}K_i(u)Z_m(u)\,du\,dv \tag{36}$$

permit the computation of the transformed potential $\tilde{\bar{\theta}}_{im}(Z)$. Thus, the dimensionless temperature potential $\theta(u,v,Z)$ can be numerically determined by truncating the expansion for a given order $m=M$ and $n=N$:

$$\theta(u,v,Z)=\sum_{i=1}^{N}\sum_{m=1}^{M}K_i(u)Z_m(v)\tilde{\bar{\theta}}_{im}(Z) \tag{37}$$

Obviously, the higher N and M, the greater the accuracy of the results.

2.3. Average temperature and the Nusselt number

The dimensionless average temperature is given by

$$\theta_{av}(Z) = \frac{1}{\pi\alpha\beta}\int_{\Omega} \theta(X,Y,Z)U(X,Y)\,\mathrm{d}\Omega \tag{38}$$

Therefore, in the plane (u, v), θ_{av} is represented by

$$\theta_{av}(Z) = \frac{1}{\pi\alpha\beta}\int_0^{\pi/2}\int_0^{v_0} \theta(u,v,Z)U(u,v)J\left(\frac{X,Y}{u,v}\right)\mathrm{d}v\,\mathrm{d}u \tag{39}$$

The Nusselt number calculated through the condition of the wall heat flux is given by

$$Nu(z) = \frac{h(z)D_h}{k} \tag{40}$$

with $h(z)$

$$h(z) = \frac{q''(z)}{T_w - T_{av}(z)} \tag{41}$$

$$q''(z) = \frac{1}{P}\int_{\Gamma} -k\frac{\partial T}{\partial \eta}\bigg|_{\Gamma}\mathrm{d}\Gamma \tag{42}$$

where, P is the perimeter of the elliptical contour. Thus, the Nusselt number is determined by:

$$Nu(Z) = \frac{D_h}{P\theta_{av}(Z)}\int_0^{2\pi} \frac{\partial\theta(u,v,Z)}{\partial v}\bigg|_{v=v_0}\mathrm{d}u \tag{43}$$

The average Nusselt number is obtained by the integration of Eq. (43):

$$\overline{Nu}(Z) = \frac{1}{Z}\int_0^Z Nu(Z')\,\mathrm{d}Z' \tag{44}$$

The thermal entry length L_{th} is defined as the position where the local Nusselt number is 5% higher than the Nusselt number in the region where the flow is fully developed [1]. Thus,

$$L_{th} \equiv \text{positive root of } \{1.05Nu(\infty) - Nu(Z) = 0\} \tag{45}$$

3. Results and discussion

For the calculation of the transformed potential $\tilde{\tilde{\theta}}_{im}(Z)$, the expansion, given by Eq. (37), was truncated for several orders N and M. It was observed that the convergence is relatively slower in the entry region ($Z < 0.001$). For truncations after orders of $M = 25$ and $N = 25$, it was verified that the values of the Nusselt numbers calculated for $Z > 0.001$ converged around 4 digits or more for all cases analyzed. Thus, the results in this work were obtained with the truncation of the expansion in $M = 25$ and $N = 25$.

The integration involved in the calculations of the perimeter P of the ellipse, parameters B_{ijmn} and the average potential $\theta_{av}(Z)$ was numerically performed by the Gauss Method (36 points). The system of differential equations for the transformed potential $\tilde{\tilde{\theta}}_{im}(Z)$, Eq. (34), was solved using the routine DIVPAG of the IMSL Library [15]. The heat transfer parameters were obtained for several ellipsis eccentricities ($\beta/\alpha = 0.1$, 0.3, 0.5, 0.7, 0.9 and 1.0) and for several behavior indices ($n = 0.1$, 0.2, 0.5, 1.0, 2.0 and 5.0). For $\beta/\alpha = 1$, the cross-section geometry is the circle and the problem can be solved more easily using the cylindrical coordinate system as in Cotta and Özisik [16].

Table 1
Average temperature and local and average Nusselt numbers: ($n = 0.2$ and 0.5, $\beta/\alpha = 0.5$)

Z	$n = 0.2$			$n = 0.5$		
	$\theta_{av}(Z)$	$Nu(Z)$	$Nu_{av}(Z)$	$\theta_m(Z)$	$Nu(Z)$	$Nu_{av}(Z)$
0.0001	0.9831	28.19	43.1	0.9854	24.38	37.0
0.0002	0.9734	22.13	33.9	0.9770	19.19	29.2
0.0005	0.9521	16.10	24.6	0.9583	14.02	21.3
0.0010	0.9256	12.69	19.4	0.9350	11.10	16.8
0.0020	0.8851	10.06	15.3	0.8991	8.826	13.3
0.0050	0.7992	7.517	11.2	0.8218	6.631	9.83
0.0100	0.6984	6.191	8.99	0.7295	5.474	7.90
0.0200	0.5567	5.323	7.35	0.5970	4.699	6.47
0.0500	0.3065	4.797	5.95	0.3533	4.203	5.23
0.1000	0.1194	4.673	5.34	0.1549	4.081	4.68
0.2000	0.0185	4.652	5.00	0.0305	4.060	4.38
0.5000	0.0001	4.651	4.79	0.0002	4.060	4.19
1.0000	0.0000	4.651	4.72	0.0000	4.060	4.12

Table 2
Average temperature and local and average Nusselt numbers: ($n = 2.0$ and 5.0, $\beta/\alpha = 0.5$)

Z	$n = 2.0$			$n = 5.0$		
	$\theta_{av}(Z)$	$Nu(Z)$	$Nu_{av}(Z)$	$\theta_m(Z)$	$Nu(Z)$	$Nu_{av}(Z)$
0.0001	0.9870	21.78	33.0	0.9873	21.16	32.0
0.0002	0.9794	17.17	26.0	0.9800	16.68	25.3
0.0005	0.9627	12.57	19.1	0.9637	12.22	18.5
0.0010	0.9416	9.957	15.1	0.9432	9.682	14.6
0.0020	0.9091	7.928	11.9	0.9115	7.710	11.6
0.0050	0.8385	5.958	8.82	0.8426	5.793	8.57
0.0100	0.7535	4.909	7.09	0.7595	4.768	6.89
0.0200	0.6299	4.190	5.79	0.6382	4.061	5.63
0.0500	0.3959	3.697	4.66	0.4075	3.566	4.51
0.1000	0.1923	3.568	4.14	0.2031	3.435	4.00
0.2000	0.0465	3.545	3.85	0.0518	3.411	3.71
0.5000	0.0007	3.545	3.67	0.0009	3.411	3.53
1.0000	0.0000	3.545	3.61	0.0000	3.411	3.47

Tables 1 and 2 present results obtained for the average temperature $\theta_{av}(Z)$, local Nusselt $Nu(Z)$ and average Nusselt $Nu_{av}(Z)$ numbers as a function of the coordinate axial Z for fluids behavior index $n = 0.2$, 0.5, 2.0 and 5.0 in ducts with aspect ratio $\beta/\alpha = 0.5$.

The average temperature behavior is shown in Fig. 2 for several behavior indices n for aspect ratio $\beta/\alpha = 0.5$. Fig. 3 shows the profile of the local Nusselt number a function of the behavior indices n for aspect ratio $\beta/\alpha = 0.5$. Average Nusselt number as a function of the behavior index and aspect ratio has showed in Figs. 4–6. It may be observed that the average Nusselt is more sensitive to the variation of the aspect ratio for pseudoplastic fluids ($n < 1$). It can also be observed that the Nusselt number is smaller for dilatant fluids ($n > 1$), when

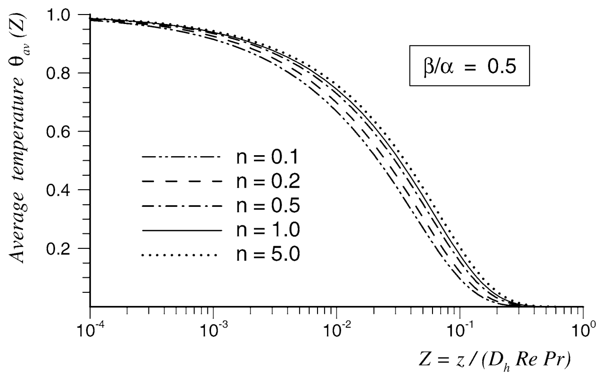

Fig. 2. Average temperature as a function of the behavior index and aspect ratio $\beta/\alpha = 0.5$.

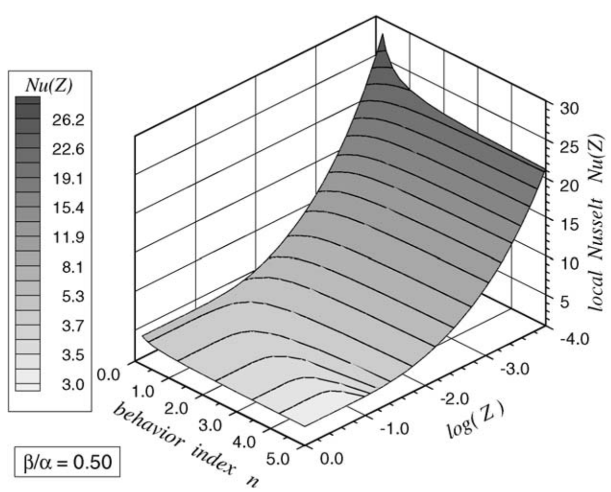

Fig. 3. Local Nusselt number as a function of the behavior index and aspect ratio $\beta/\alpha = 0.5$.

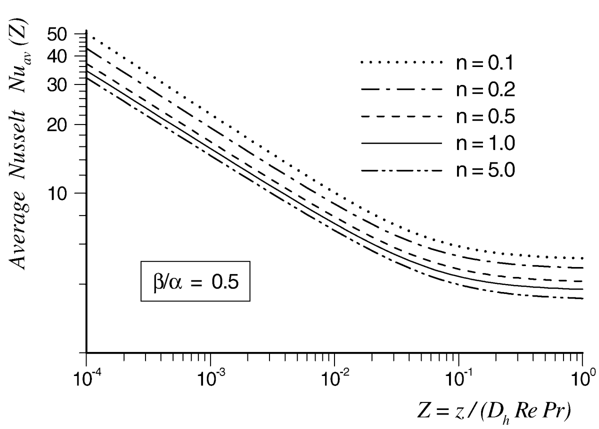

Fig. 4. Average Nusselt number along the Z-axis for several behavior indices considering $\beta/\alpha = 0.5$.

compared to pseudoplastic fluids due to the fact that the apparent viscosity increases with n for the fluids that obey the power law model.

Results obtained for the Nusselt number in the region where the flow is thermally fully developed can be seen in Tables 3 and 4. It may be observed that the thermal development is less sensitive to the variation of the flow behavior index and strongly

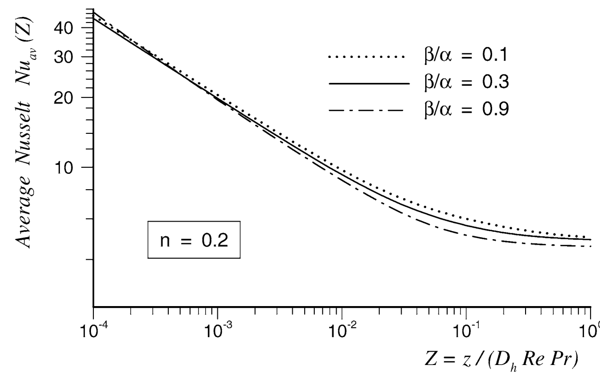

Fig. 5. Average Nusselt number along the Z-axis for several aspect ratios considering $n = 0.2$.

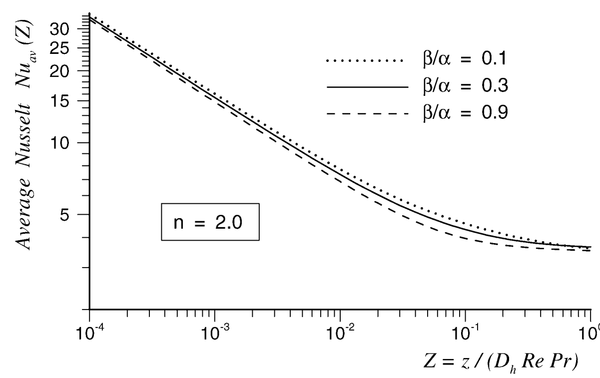

Fig. 6. Average Nusselt number along the Z-axis for several aspect ratios considering $n = 2.0$.

Table 3
Limit Nusselt number for several behavior indices n and aspect ratios β/α

n	Parameter β/α						
	0.1	0.3	0.5	0.7	0.9	1.0	$Nu(\infty)^*$
0	6.294	6.218	5.999	5.851	5.789	5.783	6.00
0.1	5.353	5.297	5.119	4.996	4.945	4.940	5.10
0.2	4.818	4.794	4.651	4.548	4.504	4.500	4.60
0.5	4.100	4.146	4.060	3.985	3.953	3.950	4.10
1.0	3.698	3.794	3.742	3.685	3.659	3.657	3.66
2.0	3.440	3.574	3.545	3.500	3.478	3.476	3.40
5.0	3.264	3.424	3.411	3.374	3.355	3.353	3.33
∞	3.135	3.314	3.314	3.284	3.266	3.264	3.30

* Results obtained by [17] for circular cross section ducts.

Table 4
Thermal entry length for several behavior indices n and aspect ratios β/α

n	Parameter β/α					
	0.1	0.3	0.5	0.7	0.9	1.0
0.1	0.148	0.0603	0.0383	0.0321	0.0305	0.0303
0.2	0.155	0.0620	0.0395	0.0327	0.0309	0.0307
0.5	0.172	0.0675	0.0418	0.0343	0.0323	0.0321
1.0	0.188	0.0722	0.0442	0.0360	0.0337	0.0335
2.0	0.202	0.0758	0.0462	0.0374	0.0347	0.0345
5.0	0.211	0.0782	0.0478	0.0384	0.0357	0.0355

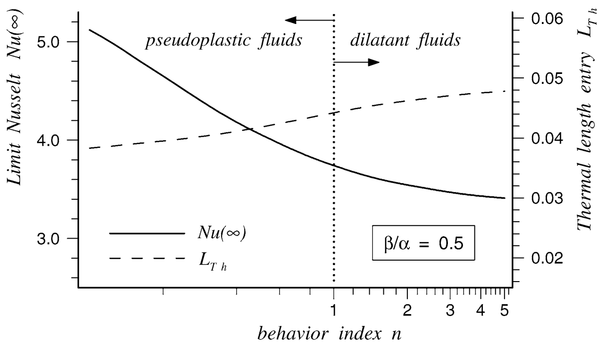

Fig. 7. Shows the Limit Nusselt and Thermal Entry Length for several behavior indices considering $\beta/\alpha = 0.5$.

dependent of the aspect ratio β/α. The thermal development diminishes with the aspect ratio, and the thermal entry length increases substantially when $\beta/\alpha \to 0$. It can also be observed that the thermal length entry is smaller when $n \to 0$. Fig. 7 show the behavior opposite of the limit Nusselt number and the thermal length entry for the pseudoplastic and dilatant fluids.

Finally, results were found in the literature only for the particular case of flow in circular cross section ducts [17]. As it can be verified, on Table 3, there is a good agreement between the results for this case.

4. Conclusions

In the present work, the heat transfer parameters for non-Newtonian fluids that follow the power law model were determined. The inherent difficulty of the application of the boundary conditions in thermal problems with this geometry was removed by means of an adequate change of variables, transforming the real domain with elliptical contour into a rectangular domain. As the energy conservation equation, in this new coordinate, system has non separable variables, the GITT was then successfully applied in order to obtain an hybrid analytical–numerical solution. The system of differential equations generated for the transformed temperature potential exhibits a slow convergence making it necessary to truncate the expansion with an order relatively high to obtain accurate results. Parameters of interest such as local, average and limit Nusselt number were obtained for several aspect ratio of the elliptical section and for several behavior indices of the fluid. The results presented here exhibit a strong dependence of the heat transfer parameters with the aspect ratio. The behavior index of the flow has a markedly influence in pseudo-plastic fluids in the region where $n \to 0$. The results obtained showed a good agreement with those found in the literature, for flow in circular cross section ducts.

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